Chapter 2
Degrees and Hamiltoneity

Obviously any pancyclic graph must contain a Hamilton cycle. In this chapter we look at some results relating the Hamiltonian property to the set of degrees of vertices in the graph.

2.1 A Theorem of Chvátal

In 1972, Chvátal [8] proved the following theorem :

Theorem 4. *Suppose G is a graph with vertices $\{a_1, a_2, \ldots, a_v\}$, $v \geq 3$, whose degrees satisfy*

$$d(a_1) \leq d(a_2) \leq \ldots, \leq d(a_v).$$

If

$$d(a_k) \leq k < \tfrac{1}{2}v \Rightarrow d(a_{v-k}) \geq v - k, \tag{2.1}$$

then G is Hamiltonian.

Proof. Suppose there is a graph G that satisfies (2.1) but is not Hamiltonian. Then G will be a subgraph of a maximally non-Hamiltonian graph, G^* say, and G^* will also satisfy (2.1).

Suppose p and q are non-adjacent vertices in G^* such that $d(p) + d(q)$ is as large as possible; for convenience, say $d(p) \leq d(q)$. Note that this implies that if r is another vertex not adjacent to q, then $d(r) \leq d(p)$.

If the edge (q, p) is added to G^*, then the resulting graph will be Hamiltonian, and every Hamilton cycle must contain (q, p) (otherwise the cycle would be in G^*); say the cycle is $(a_1, a_2, \ldots, a_v, a_1)$, where $a_1 = p$ and $a_v = q$. Define two sets S and T:

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$$S = \{i : (p, a_{i+1}) \text{ is an edge in } G^*\}, \quad T = \{i : (a_i, q) \text{ is an edge in } G^*\}.$$

Sets S and T must be disjoint: if j were a common element, then

$$(a_j, a_{j-1}, \ldots, a_1, a_{j+1}, a_{j+2}, \ldots, a_v)$$

would be a Hamilton cycle in G^*. So $S \cap T = \emptyset$, whence $|S| + |T| = |S \cup T| \leq v - 1$. Now $d(p) = |S|$ and $d(q) = |T|$, so $d(p) + d(q) = |S| + |T| < v$ and $d(p) < \frac{1}{2}v$. Moreover, as $S \cap T = \emptyset$, no a_j with $j \in S$ is adjacent to q, so $d(a_j) \leq d(p)$ for all members of S. So there are at least $d(p)$ vertices whose degrees do not exceed $d(p)$. Setting $k = d(p)$ we have $d(x_k) \leq k < \frac{1}{2}v$, so from (2.1), $d(x_{v-k}) \geq v - k$. So there are at least $k + 1$ vertices with degree at least $v - k$. As p has degree k, it cannot be adjacent to all of them. So there is a vertex, r say, with $d(r) \geq v - k$, which is not adjacent to p. But then $d(p) + d(r) \geq v > d(p) + d(q)$, contradicting the maximality of $d(p) + d(q)$. So the theorem is true by contradiction. □

If G is any graph, then a *factor* or *spanning subgraph* of G is a subgraph with vertex set $V(G)$. A *factorization* of G is a set of factors of G that are pairwise *edge-disjoint*—no two have a common edge—and whose union is all of G.

Every graph has a factorization, quite trivially: since G is a factor of itself, $\{G\}$ is a factorization of G. However, it is more interesting to consider factorizations in which the factors satisfy certain conditions. In particular a *one-factor* is a factor that is a regular graph of degree 1. In other words, a one-factor is a set of pairwise disjoint edges of G that between them contain every vertex. A *one-factorization* of G is a decomposition of the edge set of G into edge-disjoint one-factors. Similarly a *two-factor* is a factor that is a regular graph of degree 2—a union of disjoint cycles—and a *two-factorization* of G is a decomposition of the edge set of G into edge-disjoint two-factors.

2.2 A Theorem of Fan

The following theorem was proven by Geng-Hua Fan [11] in 1984:

Theorem 5. *Let G be a 2-connected graph on $n \geq 3$ vertices. If $D(x, y) = 2$ implies $\max\{d(x), d(y)\} \geq n/2$ for all vertices x and y, then G is Hamiltonian.*

Proof. The proof is by contradiction. Let G be a graph satisfying the given condition and G has no Hamilton cycle. Let $P = (v_0, v_1, \ldots, v_m)$ be a longest path in G of length m, chosen such that $d(v_0) + d(v_m)$ is as large as possible. If $d(v_0) + d(v_m) \geq n$, then there are at least two consecutive vertices on P, say v_i and v_{i+1}, such that $v_i v_m, v_{i+1} v_0 \in E(G)$. This gives us a cycle of length $m + 1$. Since G is 2-connected, we have either a Hamilton cycle or a path of length $m + 1$. Both lead to contradictions. Hence, $d(v_0) + d(v_m) < n$. Without loss of generality, we may assume $d(v_0) < n/2$.

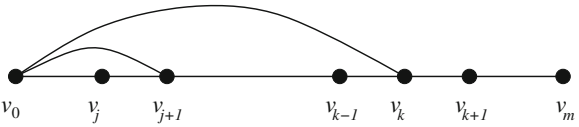


Fig. 2.1 Case where k is as large as possible

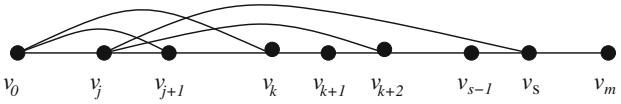


Fig. 2.2 Case where $s \geq k+2$

An argument similar to that described above shows that:

Claim 1. G has no cycle of length $m+1$.

Since G is 2-connected, v_0 is joined to at least one vertex on P other than v_1, say v_k. We choose k as large as possible (see Fig. 2.1). Clearly, $k \geq 2$ and $k < m$, otherwise we have a cycle of length $m+1$, which contradicts Claim 1.

Claim 2. $v_0v_i \in E(G)$ for all $1 \leq i \leq k$.

Proof. As $v_0v_k \in E(G)$, there is another longest path $(v_{k-1}, v_{k-2}, \ldots, v_0, v_k, \ldots, v_m)$ in G. By the maximality of $d(v_0) + d(v_m)$, $d(v_{k-1}) < d(v_0) < n/2$. So $\max\{d(v_{k-1}), d(v_0)\} < n/2$. Hence, $d(v_0, v_{k-1}) \neq 2$. However, (v_0, v_k, v_{k-1}) is a path of length 2 and thus we must have $v_0v_{k-1} \in E(G)$. If $k-1=1$ we stop, if not we repeat this process and thus we obtain that $v_0v_i \in E(G)$ for all $1 \leq i \leq k$.

Claim 3. $d(v_i) \leq d(v_0)$ for all $1 \leq i \leq k-1$.

Proof. Suppose that $d(v_j) > d(v_0)$ for some $1 \leq j \leq k-1$. Since $v_0v_{j+1} \in E(G)$, by Claim 2, $(v_j, v_{j-1}, \ldots, v_0, v_{j+1}, \ldots, v_k, \ldots, v_m)$ is another longest path with $d(v_j) + d(v_m) > d(v_0) + d(v_m)$, a contradiction.

Claim 4. $d(v_{k+1}) > d(v_0)$.

Proof. Since $v_0v_{k+1} \in E(G)$ by the choice of v_k and the path (v_0, v_k, v_{k+1}) is of length 2, it follows that $d(v_0, v_{k+1}) = 2$. Hence, $\max\{d(v_0), d(v_{k+1})\} \geq n/2$. So $d(v_{k+1}) \geq n/2 > d(v_0)$.

For every $1 \leq i \leq k-1$, the vertex v_i cannot be joined to any vertex outside P by Claim 2. Since G is 2-connected, it follows that there exists an edge v_jv_s in G such that $j < k < s$. We choose v_jv_s in G such that $j < k < s$ and s is as large as possible. We now consider two cases.

Case 1. $s \geq k+2$ (see Fig. 2.2). By Claim 2, $v_0v_{j+1} \in E(G)$ and so there is a longest path $(v_{s-1}, v_{s-2}, \ldots, v_{j+1}, v_0, \ldots, v_j, v_s, \ldots, v_m)$ in G. It follows that $d(v_{s-1}) \leq d(v_0) < n/2$ by the maximality of $d(v_0) + d(v_m)$. On the other hand, $d(v_j) \leq d(v_0) < n/2$ by Claim 3. Therefore $\max\{d(v_{s-1}), d(v_j)\} < n/2$. Hence, $d(v_j, v_{s-1}) \neq 2$. But (v_j, v_s, v_{s-1}) is a path of length 2 and thus we

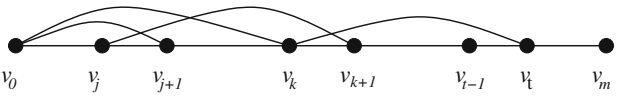


Fig. 2.3 Case where $s = k + 1$

must have that $v_j v_{s-1} \in E(G)$. If $s - 1 > j + 1$, we have another longest path $(v_{s-2}, v_{s-3}, \ldots, v_{j+1}, v_0, \ldots, v_j, v_{s-1}, v_s, \ldots, v_m)$ in G. Repeat this process to obtain $v_j v_{s-2} \in E(G)$. If $s - 2 > j + 1$ we go through the same process again. Consequently, we have $v_j v_i \in E(G)$ for all $j + 1 < i < s$. In particular, $v_j v_{k+2} \in E(G)$ because $s \geq k + 2$. This means that there is a longest path

$$(v_{k+1}, v_k, \ldots, v_{j+1}, v_0, \ldots, v_j, v_{k+2}, \ldots, v_m)$$

in G with $d(v_{k+1}) + d(v_m) > d(v_0) + d(v_m)$, a contradiction.

Case 2. $s = k + 1$ (see Fig. 2.3). Note that $(v_k, v_{k-1}, \ldots, v_{j+1}, v_0, \ldots, v_j, v_{k+1}, \ldots, v_m)$ is a longest path and so by the maximality of $d(v_0) + d(v_m)$ we obtain $d(v_k) \leq d(v_0) < n/2$.

If $k + 1 = m$, we obtain a cycle $(v_0, v_1, \ldots, v_j, v_{k+1}, v_k, \ldots, v_{j+1}, v_0)$ of length $m + 1$. This contradicts Claim 1. Now assume $k + 1 < m$. It follows from the 2-connectedness of G and the choice of v_s that there must be $v_k v_t \in E(G)$ such that $t \geq k + 2$. This implies that there is another longest path $(v_{t-1}, v_{t-2}, \ldots, v_{k+1}, v_j, \ldots, v_0, v_{j+1}, \ldots, v_k, v_t, \ldots, v_m)$ in G. By the maximality of $d(v_0) + d(v_m)$ we must have $d(v_{t-1}) \leq d(v_0) < n/2$. Hence, $\max\{d(v_k), d(v_{t-1})\} < n/2$. Therefore $d(v_k, v_{t-1}) \neq 2$. Now note that (v_k, v_t, v_{t-1}) is a path of length 2. So we obtain $v_k v_{t-1} \in E(G)$. If $t - 1 > k + 1$ we repeat this process to obtain $v_k v_t \in E(G)$ for all $k + 1 \leq i \leq t$. In particular, $v_k v_{k+2} \in E(G)$ because $t \geq k + 2$. Hence, there is a longest path $(v_{k+1}, v_j, \ldots, v_0, v_{j+1}, \ldots, v_k, v_{k+2}, \ldots, v_t, \ldots, v_m)$ in G. By Claim 4, $d(v_{k+1}) + d(v_m) > d(v_0) + d(v_m)$, a contradiction. This completes the proof. □

2.3 A Theorem of Bondy and Its Generalization

The following result is due to Bondy [5]:

Theorem 6. *Let G be a 2-connected graph on n vertices. Suppose that for every set of three independent vertices x, y, and z, we have $d(x) + d(y) + d(z) \geq m \geq n + 2$. Then G contains a cycle of length at least* $\min\{n, 2m/3\}$.

In [5], Bondy conjectured that the same result is true without the condition $m \geq n + 2$ and can be generalized to a k-connected graph for $k \geq 3$. In Theorem 7, below, we present a proof by Fournier and Fraisse [12] of this conjecture. Consequently Theorem 6 holds, and we do not need to present a proof of it.

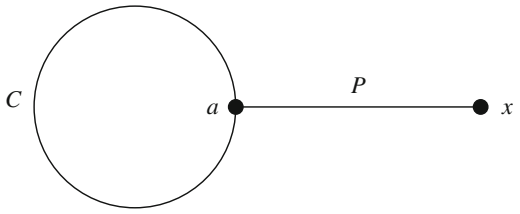


Fig. 2.4 P with only one point (an endpoint) on C

Suppose G is a k-connected graph on n vertices, $k \geq 2$, such that the degree-sum of any $k+1$ independent vertices is at least m. We prove G contains a cycle of length at least $\min\{n, 2m/(k+1)\}$. Several lemmas are involved.

We shall use the following notation:

1. Consider a cycle C in G for which an orientation has been chosen. If a and b are vertices of C, $[a, b]_C$ denotes the subgraph of C which is the path with endpoints a and b whose vertices are met on C when one goes on C from a to b.
2. For a vertex a of C, a^+ is the vertex on C immediately after a and (a^-) is the vertex just before a. Similarly, $a^{+\ell}$ $(a^{-\ell})$ is the ℓth vertex on C after (before) a. If S is a set of vertices of C, then $S^+ = \{a^+ \mid a \in S\}$.
3. If P is a path in G and if a and b are two vertices of P, $[a, b]_P$ denotes the subgraph of P which is the path with endpoints a and b.
4. If H is a subgraph of G and if a is a vertex of G, $\Gamma_H(a)$ is the set of neighbors of a in H, and $d_H(a) = |\Gamma_H(a)|$.
5. A C-path is a path of length at least two in G such that only its endpoints are on C.

Let C be a cycle of G of maximum length. We assume that C is not a Hamilton cycle. Let $R = G \setminus C$. Give an arbitrary orientation to C.

The proof of the following lemma can also be found in [9]:

Lemma 7.1. *Let P be a path of maximal length among all the paths with a given endpoint a on C and all the other vertices not on C (see Fig. 2.4). Let x be the other endpoint of P. Then there exists a C-path which contains x and all its neighbors in R such that one of its endpoints on C is a.*

Proof. If $d_R(x) = 0$ or 1, the result immediately follows from the fact that G is 2-connected. Assume $d_R(x) \geq 2$. Since P is of maximum length, all the neighbors of x in R are on P. Let x' be the furthest neighbor of x in R on P (see Fig. 2.5). Let C' be the cycle $[x, x']_P \cup \{xx'\}$. Let P_1 and P_2 be two disjoint paths between C and C' and let $Q = [x', a]_P$.

We now construct two disjoint paths P_3 and P_4 between C and C' such that x' is the endpoint of P_3 on C' and a is the endpoint of P_3 or P_4 on C.

Case 1. P_1 (respectively, P_2) does not intersect Q. Set $P_3 = Q$ and $P_4 = P_1$ (respectively, P_2).

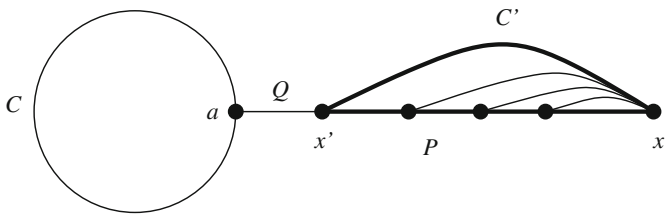


Fig. 2.5 x' is the furthest neighbor of x

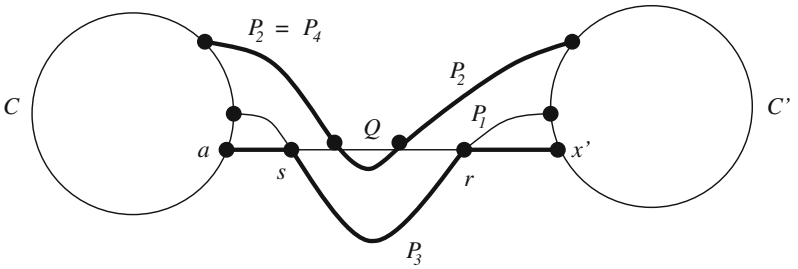


Fig. 2.6 r and s are on the same path

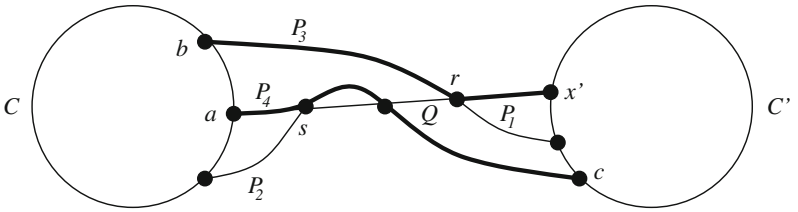


Fig. 2.7 r and s are on different paths

Case 2. P_1 and P_2 intersect Q. Let r be the vertex nearest to x' along Q and s be the vertex nearest to a along Q.

1. If r and s are on the same path P_1 (see, for example, Fig. 2.6), then

$$P_3 = [x', r]_Q \cup [r, s]_{P_1} \cup [s, a]_Q \text{ and } P_4 = P_2.$$

2. If $r \in P_1$ and $s \in P_2$, write b for the endpoint of P_1 on C and c for the endpoint of P_2 on C'. We set (see Fig. 2.7):

$$P_3 = [x', r]_Q \cup [r, b]_{P_1} \text{ and } P_4 = [c, s]_{P_2} \cup [s, a]_Q.$$

Let y be the endpoint of P_4 on C'. We orient C' so that $x \in [x', y]_{C'}$. We now have two possible cases:

1. If all the neighbors of x in P are on $[x', y]_{C'}$, we easily construct the C-path as required (see Fig. 2.8).

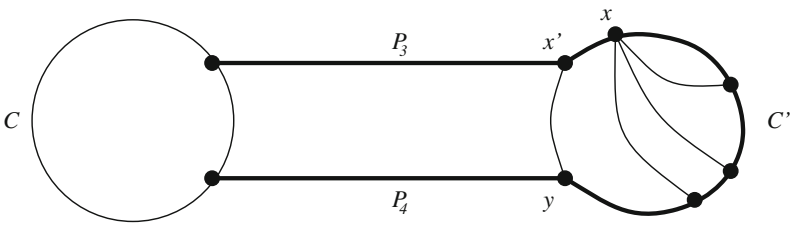


Fig. 2.8 The C-path in case 1

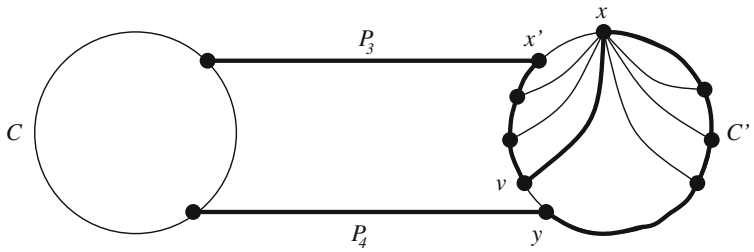


Fig. 2.9 The C-path in case 2

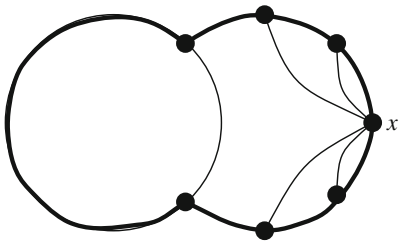


Fig. 2.10 The required new cycle

2. Suppose x has at least one neighbor on $[y^+, x'^-]_{C'}$. Let v be the neighbor of x on $[y^+, x'^-]_{C'}$ nearest y along C'. The C-path (see Fig. 2.9)

$$P_3 \cup [x', v]_{C'} \cup \{vx\} \cup [x, y]_{C'} \cup P_4$$

contains x and all its neighbors in R. □

Lemma 7.2. *Suppose a, P, and x are defined as in Lemma 7.1. Then $d_R(x) \leq \frac{|C|-4}{2}$.*

Proof. Use the C-path obtained in Lemma 7.1 to construct (see Fig. 2.10) a new cycle of length at least $\frac{|C|+2}{2} + d_R(x) + 1$. Hence, $(|C| + 2)/2 + d_R(x) + 1 \leq |C|$ and the result follows. □

Lemma 7.3. *Let a, P, and x be as in Lemma 7.1. Then $d(x) \leq |C|/2$.*

Proof. Since the cycle C is of maximum length, it follows that x is not adjacent to the vertex $a^{+\ell}$ if $1 \le \ell \le \min\{|P|-1, |C|-1\}$. We now have

1. $|P|-1 \ge d_R(x)+1$ by the choice of P;
2. $|C|-1 \ge 2d_R(x)+3$ by Lemma 7.2.

Thus x is not adjacent to $a^{+\ell}$ for $1 \le \ell \le d_R(x)+1$. Similarly, x is not adjacent to $a^{-\ell}$ for $1 \le \ell \le d_R(x)+1$.

Now by Lemma 7.2

$$2(d_R(x)+1) \le 2\left(\frac{|C|-4}{2}+1\right) \le |C|-2.$$

Therefore the set of vertices $\{a^{+\ell} \mid 1 \le \ell \le d_R(x)+1\}$ cannot intersect the set of vertices $\{a^{-\ell} \mid 1 \le \ell \le d_R(x)+1\}$. In addition, since C is of maximum length, it follows that no two consecutive vertices of C are simultaneously adjacent to x. Hence,

$$d_C(x) \le 1 + \frac{|C|-1-2(d_R(x)+1)+1}{2} \le \frac{|C|}{2} - d_R(x).$$

Therefore $d(x) \le |C|/2$, as required. □

Lemma 7.4. *For any C-path with endpoints a_1 and a_2 on C, which is not reduced to an edge,*

$$d_C(a_1^+) + d_C(a_2^+) \le |C|.$$

Proof. Since C is of maximal length,

$$\Gamma_C^+(a_1^+) \cap \Gamma_C(a_2^+) \cap [a_2^{++}, a_1] = \emptyset$$

(see Fig. 2.11). Thus

(1) $$d_{[a_2^+, a_1^-]}(a_1^+) + d_{[a_2^{++}, a_1]}(a_2^+) \le |[a_2^{++}, a_1]|.$$

Similarly,

(2) $$d_{[a_1^{++}, a_2]}(a_1^+) + d_{[a_1^+, a_2^-]}(a_2^+) \le |[a_1^{++}, a_2]|.$$

Now add the inequalities (1) and (2) to obtain $d_C(a_1^+) + d_C(a_2^+) \le |C|$. □

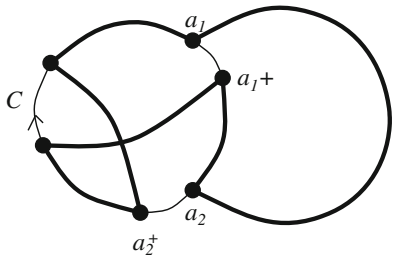


Fig. 2.11 A C-path with a_1 and a_2 on C

Lemma 7.5. *Define a, P, and x as in Lemma 7.1. Then* $d(x) + d_C(a^+) \leq |C|$.

Proof. We consider two cases.

Case 1. $d_{C\setminus\{a\}}(x) = 0$. By Lemma 7.1, there exists a C-path which contains x and all its neighbors in R such that one of its endpoints on C is a. Let b be the other endpoint of this C-path. The vertex a^+ is not adjacent to $b^{+\ell}$ if $1 \leq \ell \leq d_R(x) + 1$ because C is of maximum length (see Fig. 2.12). Therefore $d_C(a^+) \leq |C| - (d_R(x) + 1)$, that is, $d_C(a^+) + d_R(x) \leq |C| - 1$. Now since $d_C(x) \leq 1$, it follows that $d_C(a^+) + d(x) \leq |C|$.
Case 2. $d_{C\setminus\{a\}}(x) \geq 1$. Let b be the neighbor of x on $C \setminus \{a\}$ such that $d_{[b^+,a^-]_C}(x) = 0$. Then $|[b^+, a^-]_C| \geq d_R(x) + 1$. In addition, a^+ is not adjacent to any of the vertices $b^{+\ell}$ for $1 \leq \ell \leq d_R(x) + 1$ (see Fig. 2.13).

On the other hand, let v be any neighbor of x on C, different from a and b. Then $v \notin [b, a]_C$. So $v^+ \notin \{b^{+\ell} \mid 1 \leq \ell \leq d_R(x) + 1\}$ and a^+ is not adjacent to v^+ (see Fig. 2.14).

Finally, since a^+ is not adjacent to itself,

$$\begin{aligned} d_C(a^+) &\leq |C| - 1 - (d_R(x) + 1) - (d_C(x) - 2) \\ &\leq |C| - d_R(x) - d_C(x). \end{aligned}$$

Therefore $d(x) + d_C(a^+) \leq |C|$, as required. □

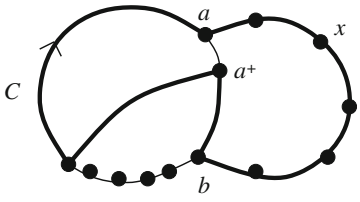


Fig. 2.12 Case 1: $d_{C\setminus\{a\}}(x) = 0$

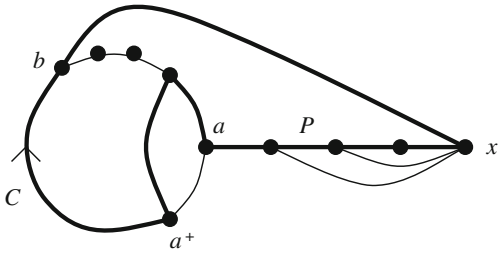


Fig. 2.13 Case 2: $d_{C\setminus\{a\}}(x) \geq 1$ and a^+ is not adjacent to $b^{+\ell}$

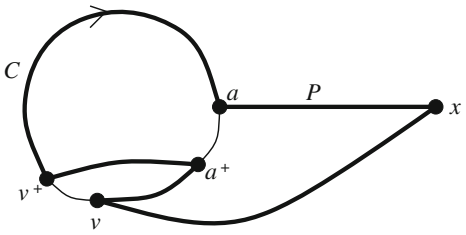


Fig. 2.14 Case 2: $d_{C\setminus\{a\}}(x) \geq 1$ and a^+ is not adjacent to v^+

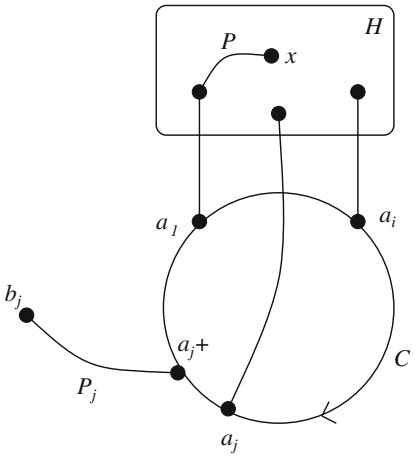


Fig. 2.15 k parallel edges between H and C

We are now ready to prove the following theorem of Fournier and Fraisse [12]:

Theorem 7. *Let G be a k-connected graph where $k \geq 2$, such that the degree-sum of any $k+1$ independent vertices is at least m. Then G contains a cycle of length at least* $\min\{n, 2m/(k+1)\}$.

Proof. Let a_1 be a vertex of C such that $d_R(a_1^+)$ is minimum. Suppose that P is a path with an endpoint a_1 such that $V(P) \setminus \{a_1\} \subset R$ and of maximum length. Let x be the other endpoint of P in R and let H be the component of x in R.

Since G is k-connected, there exist k edges between H and C with distinct endpoints on C and we can suppose that one of these edges is the edge of P containing a_1 as an endpoint. Note that these edges do not necessarily have distinct endpoints in H. We denote the endpoints of these edges on C by $a_1, a_2, \ldots, a_k$ (see Fig. 2.15).

Define

$$I = \{i \in \{1, 2, \ldots, k\} \mid d_R(a_i^+) = 0\}$$

and

$$J = \{j \in \{1, 2, \ldots, k\} \mid d_R(a_j^+) \neq 0\}.$$

Let $P_j, j \in J$, be a path starting from a_j^+ such that $v(P_j) \setminus \{a_j^*\} \subset R$ and of maximum length. Assume b_j is the endpoint of P_j in R. Then

1. P_j does not intersect H (and P) because there is no C-path between a_j and a_j^+.
2. If $j_1 \neq j_2$, then P_{j_1} does not intersect P_{j_2} because it is impossible to have a C-path between a_{j_1} and a_{j_2} through H and also a distinct C-path between $a_{j_1}^+$ and $a_{j_2}^+$.
3. The set $\{x\} \cup \{a_i^+ \mid i \in I\} \cup \{b_j \mid j \in J\}$ is an independent set.
4. $d(x) \leq |C|/2$ by Lemma 7.3.
5. $d(b_j) \leq |C|/2$ for $j \in J$ by Lemma 7.3.

We now consider three cases.

Case 1. $|I| \geq 2$. Let $p = |I|$ and $I = \{i_1, i_2, \ldots, i_p\}$. Then $d_R(a_{i_1}^+) = d_R(a_{i_2}^+) = \ldots = d_R(a_{i_p}^+) = 0$. By Lemma 7.4,

$$\begin{aligned} d_C(a_{i_1}^+) + d_C(a_{i_2}^+) &\leq |C| \\ d_C(a_{i_2}^+) + d_C(a_{i_3}^+) &\leq |C| \\ &\vdots \\ d_C(a_{i_{p-1}}^+) + d_C(a_{i_p}^+) &\leq |C| \\ d_C(a_{i_p}^+) + d_C(a_{i_1}^+) &\leq |C|, \end{aligned}$$

so $\sum_{i \in I} d(a_i^+) \leq p(|C|/2)$. Therefore

$$m \leq d(x) + \sum_{i \in I} d(a_i^+) + \sum_{j \in J} d(b_j) \leq (k+1)\frac{|C|}{2}.$$

Hence, $|C| \geq \frac{2m}{k+1}$, as required.

Case 2. $|I| = 1$. From the choice of a_1 we must have $I = \{1\}$. Now by Lemma 7.5, $d_C(a_i^+) + d(x) \leq |C|$. In addition, since $d_R(a_i^+) = 0$, it follows that

$$m \leq d(x) + d(a_1^+) + \sum_{j=2}^{k} d(b_j) \leq (k+1)\frac{|C|}{2}.$$

Hence, $|C| \geq \frac{2m}{k+1}$.

Case 3. $|I| = 0$. In this case we have

$$m \leq d(x) + \sum_{j=1}^{k} d(b_j) \leq (k+1)\frac{|C|}{2}.$$

Hence, $|C| \geq \frac{2m}{k+1}$. This completes the proof. □

The following theorem is an immediate result of Theorem 7 with $k = 2$:

Theorem 8 ([5]). *Let G be a 2-connected graph on n vertices. Suppose that for every set of three independent vertices x, y, and z, we have $d(x) + d(y) + d(z) \geq 3n/2 - 1$. Then G is Hamiltonian.*



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